Laser Lecture 5 Three Level Laser System

Some results from previous lectures/notes to be used in th present lecture/note

$$\alpha = \frac{\pi^2 c^2}{n_o^2 \omega^2} \frac{1}{t_{sp}} g(\omega)\ (N_1 - N_2)$$

$$N_2 - N_1 \geq \frac{4\nu^2 n_o^3}{c^3} \frac{t_{sp}}{t_c} \frac{1}{g(\omega)}$$

$$\Gamma_{12} = N_1 \frac{\pi^2 c^3}{\hbar\, n_o^3 \omega'^3} \frac{1}{t_{sp}} g(\omega')u$$

Consider a two level laser system having energy levels E_1 and E_2 with N_1 and N_2 number of atoms per unit volume, respectively

A radiation with frequency ω and energy density u is incident on the system

Atoms absorb the radiation and are excited to the upper level. The number of atoms per unit volume per unit time absorbing the radiation will be

$$\Gamma_{12} = \frac{\pi^2 c^3}{\hbar\omega^3\, n_o^3}\frac{1}{t_{sp}} u\, g(\omega)\, N_1 = W_{12} N_1 \qquad W_{12} = \frac{\pi^2 c^3}{\hbar\omega^3\, n_o^3}\frac{1}{t_{sp}} u\, g(\omega)$$

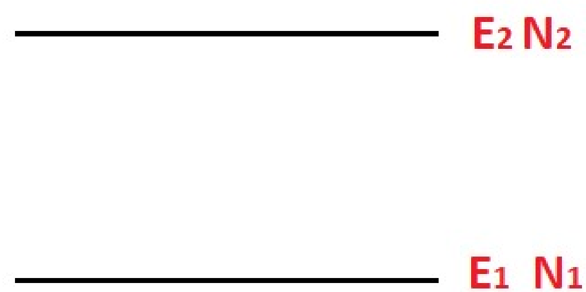
Number of atoms undergoing stimulated emission per unit volume per second

$$\Gamma_{21} = W_{21} N_2 = W_{12} N_2$$

Atoms in E_2 will also undergo spontaneous transitions from E_2 to E_1

Let A_{21} and S_{21} represent the radiative and non-radiative transition rates from E_2 to E_1 then the number of atoms per unit volume undergoing transitions from E_2 to E_1 per second

$$T_{21} = A_{21} + S_{21}$$



The rate of change of population of E_2 and E_1

$$\frac{dN_2}{dt} = W_{12}(N_1 - N_2) - T_{21} N_2$$

$$\frac{dN_1}{dt} = -W_{12}(N_1 - N_2) + T_{21} N_2$$

$$\frac{d}{dt}(N_1 + N_2) = 0 \qquad\qquad N_1 + N_2 = constant = N$$

i.e. the total number of atoms per unit volume is constant

At steady state

$$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt} \qquad\qquad W_{12}(N_1 - N_2) = T_{21}N_2$$

$$\frac{N_2}{N_1} = \frac{W_{12}}{W_{12} + T_{21}}$$

Both W_{12} and T_{21} are positive $\rightarrow N_2 < N_1$.

This shows that we can never achieve a steady state population inversion by optical pumping between just two levels.

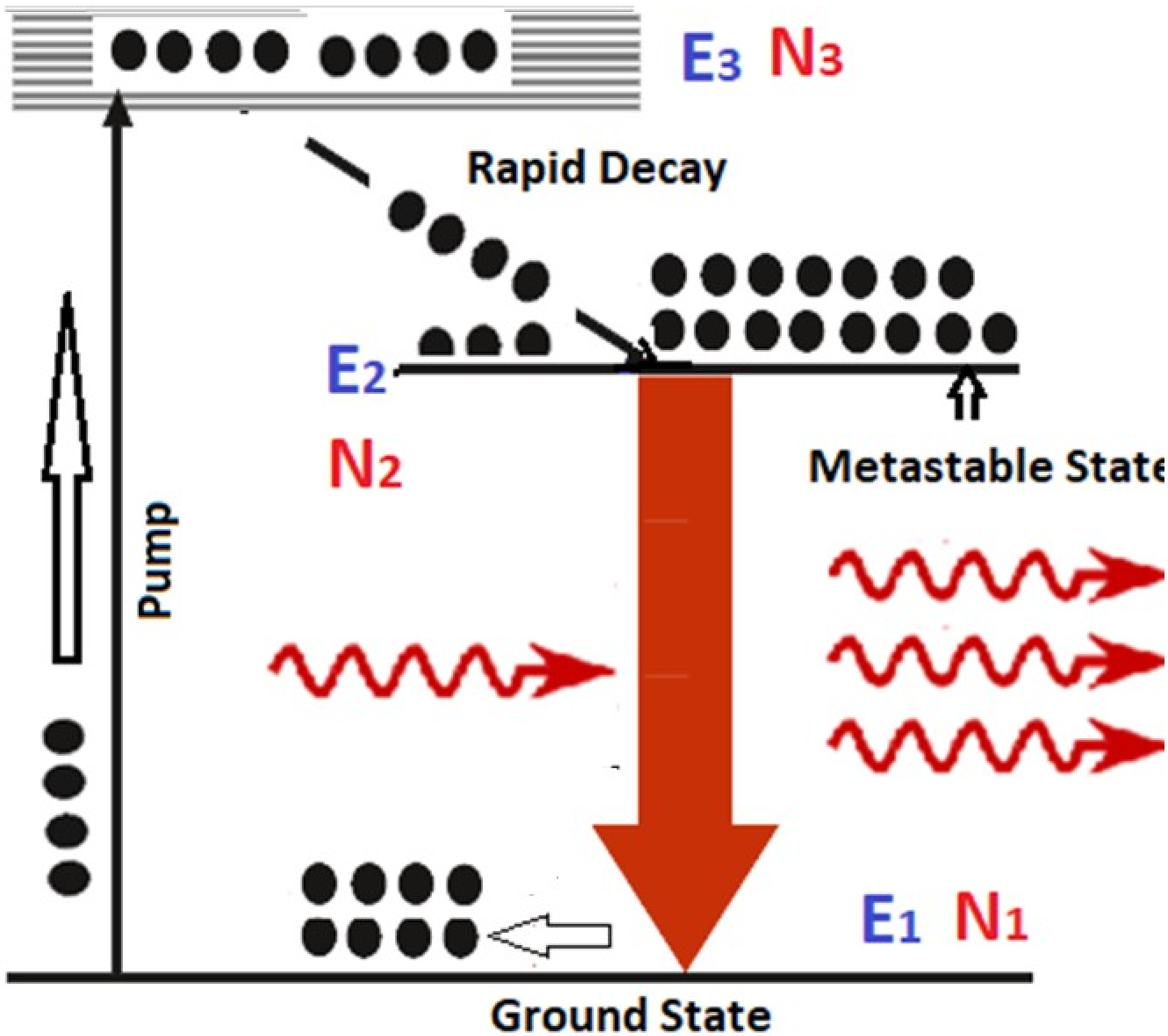
Three Level Laser System

Consider three level laser system with energy levels E_1, E_2 and E_3. All these energy levels are degenerate.

N_1, N_2 and N_3 are population densities of the three energy levels, respectively.

The pump is assumed to lift atoms from level 1 to level 3 from which they decay rapidly to level 2 through non-radiative decay.

Upper level 3 is not a laser level, it can be broad level (or a group of broad levels) so that a broadband light source may be used as a pump.

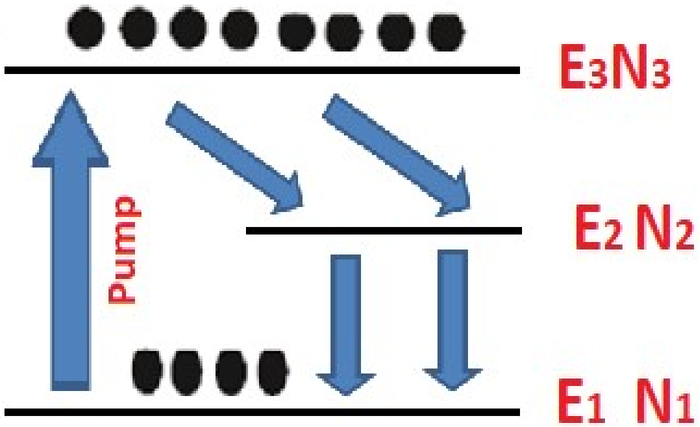

E3 N3
Rapid Decay
E2
N2
Metastable State
Pump
E1 N1
Ground State

Rate equations

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - T_{32}N_2 \qquad (1)$$

W_p= rate of pumping per atom from energy level 1 to 3

$T_{32} = A_{32} + S_{32}$ $\qquad$ $T_{31}N_3$ has been neglected



$$\frac{dN_2}{dt} = W_1(N_1 - N_2) + T_{32}N_3 - T_{21}N_2 \qquad (2)$$

$$\frac{dN_1}{dt} = W_p(N_3 - N_1) - W_1(N_1 - N_2) + T_{21}N_2 \qquad (3)$$

$$W_1 = \frac{\pi^2 c^3}{\hbar\omega^3 n_o^2} A_{21}\ g_1(\omega) I_1 \qquad (4)$$

W_1= rate of stimulated transition between levels 1 and 2

I_1= intensity of radiation in the transition 2 → 1

$g_1(\omega)$= line shape function describing the transitions between levels 1 and 2

For good laser action $A_{21} >> S_{21}$

At steady state $$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt} = \frac{dN_3}{dt} \qquad (5)$$

From equation (1) $W_p(N_1 - N_3) = T_{32} N_3$

$$N_3 = \frac{W_p}{W_p + T_{32}} N_1 \qquad (6)$$

From equations (2), (5) and (6)

$$W_1 N_1 - W_1 N_2 + \frac{W_p T_{32}}{W_p + T_{32}} N_1 = N_2 T_{21} \qquad (7)$$

$$N_2 = \frac{W_1(W_p + T_{32}) + W_p T_{32}}{(W_p + T_{32})(W_1 + T_{21})} N_1 \qquad (8)$$

$$N = N_1 + N_2 + N_3 \qquad (9)$$

$$\frac{N_2 - N_1}{N} = \frac{W_p(T_{32} - T_{21}) - T_{32}T_{21}}{3W_pW_1 + 2W_pT_{21} + 2T_{32}W_1 + T_{32}W_p + T_{32}T_{21}} \qquad (10)$$

From (10) to obtain population inversion between levels 2 and 1, N_2-N_1 should be positive. For this necessary (but not sufficient) condition is that $T_{32} >> T_{21}$ i.e. lifetime of level 3 must be smaller than the lifetime of level 2.

When this condition is satisfied, the minimum pumping rate required to achieve population inversion

$$W_{pt}(T_{32} - T_{21}) - T_{32}T_{21} = 0 \qquad W_{pt} = \frac{T_{32}T_{21}}{T_{32} - T_{21}} \qquad (11)$$

For $T_{32} >> T_{21}$ $\qquad W_{pt} \approx T_{21} \qquad (12)$

Under the same approximation

$$\frac{N_2 - N_1}{N} = \frac{W_p(T_{32} - \cancel{T_{21}}) - T_{32}T_{21}}{(3W_p + 2T_{32})W_1 + \cancel{2W_pT_{21}} + (W_p + T_{21})T_{32}}$$

$$= \frac{(W_p - T_{21})T_{32}}{(3W_p + 2T_{32})W_1 + (W_p + T_{21})T_{32}} \qquad (13)$$

Dividing by $\dfrac{T_{32}}{W_p + T_{21}}$

$$\frac{N_2 - N_1}{N} = \frac{(W_p - T_{21}) \big/ (W_p + T_{21})}{[1 + \dfrac{3W_p + 2T_{32}}{T_{32}(W_p + T_{21})} W_1]} \qquad (14)$$

Below threshold

$$\frac{N_2 - N_1}{N} = \frac{W_p - T_{21}}{W_p + T_{21}} \qquad (15)$$

Thus when W_1 is small i.e. when the intensity of radiation corresponding to the laser transition is small [eqn (4)] then the population inversion is independent of I_1, and thus there is an exponential amplification of the beam.

$$W_1 = \frac{\pi^2 c^3}{\hbar \omega^3 n_o^2} A_{21}\ g_1(\omega) I_1 \qquad (4)$$

As the laser starts oscillating W_1 becomes large and from eqn (14) this reduces the inversion N_2-N_1 which in turn reduces the amplification.

When laser oscillates under steady state condition, the intensity of radiation at the laser transition increases to such a value that the value of N_2-N_1 is the same as the threshold value.

For a population inversion N_2-N_1 the gain coefficient of the laser system is

$$\alpha = -\frac{\pi^2 c^2}{n_o^2 \omega^2} \frac{1}{t_{sp}} g(\omega) (N_2 - N_1)$$

$$\alpha = \frac{\alpha_o}{1 + \frac{3W_p + 2T_{32}}{T_{32}(W_p + T_{21})} W_1}$$

Here

$$\alpha_o = -\frac{\pi^2 c^2}{n_o^2 \omega^2} \frac{1}{t_{sp}} g(\omega) N \frac{W_p - T_{21}}{W_p + T_{21}}$$

$$\alpha = \frac{\alpha_o}{1 + (I_1/I_s)\tilde{g}(\omega)}$$

I_s is the saturation intensity.

$$I_s = \frac{\hbar \omega^3 n_o^2}{\pi^2 c^2 A_{21} g(\omega_o)} \frac{T_{32}(W_p + T_{21})}{(3W_p + 2T_{32})}$$

Here, we have introduced a line shape function $\tilde{g}(\omega)$ which is normalized so that it can have value of unity at $\omega=\omega_o$, the centre of the line

$$\tilde{g}(\omega) = \frac{g(\omega)}{g(\omega_o)}$$

Since $g(\omega) \leq g(\omega_o)$ for all ω, $0<\tilde{g}(\omega)<1$

Consider a monochromatic wave at frequency ω_o interacting with the two laser levels. $\tilde{g}(\omega) = 1$.

For $I << I_s$, ΔN is independent of the intensity of the incident radiation

For $I \approx I_s$, ΔN is a function of I.

For $I = I_s$, ΔN is half its value at low intensities

Atoms are pumped from energy level E_1 to E_3. Atoms make transition from level E_3 to E_2. Energy level E_2 is the metastable state (having life time 10^{-5} to 10^{-3} seconds). Thus, population of atoms become more in the energy state E_2 as compared to E_1.

When external photons having frequency $\omega=[(E_2-E_1)/\hbar]$ are allowed to interact with amplifying medium in the excited state, the stimulated emission occurs between E_2 and E_1 producing laser. E_2 is known as upper laser level and E_1 is known as lower laser level.

An example of a three-level laser medium is ruby (Cr^{3+}: Al_2O_3), as used by Maiman for the first laser.

Pure three-level laser gain media are seldom used, while quasi-three-level media (see below) are quite common, particularly in the context of fiber lasers.

A quasi-three-level laser medium is a kind of intermediate situation, where in the lower laser level is so close to the ground state that an appreciable population in that level occurs at thermal equilibrium. This unpumped gain medium causes some reabsorption loss at the laser wavelength, and transition is reached only for some finite pump power. For higher pump powers, there is gain, as required for laser operation.

Examples of quasi-three-level media are all ytterbium-doped gain media (e.g. Yb:YAG, or Yb:glass as used in optical fibers), neodymium-doped media operated on the ground state transition (e.g. 946 nm for Nd:YAG), thulium-doped crystals and glasses for 2-µm emission, and erbium-doped media for 1.5 or 1.6-µm emission, such as erbium-doped fiber amplifiers.

https://www.rp-photonics.com/four_level_and_three_level_gain_media.html#:~:text=An%20example%20of%20a%20three-level%20laser%20medium%20is,particularly%20in%20the%20context%20of%20fiber%20lasers%20.

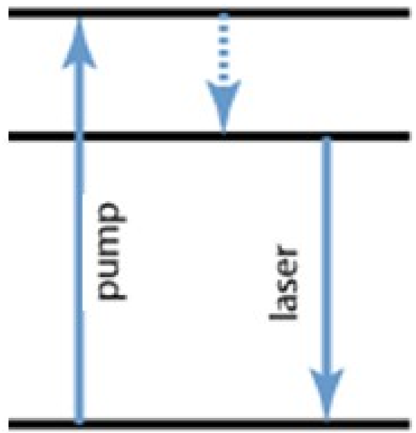


Figure 1

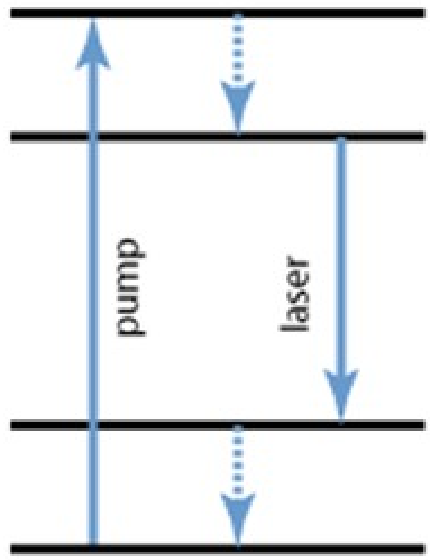


Figure 2

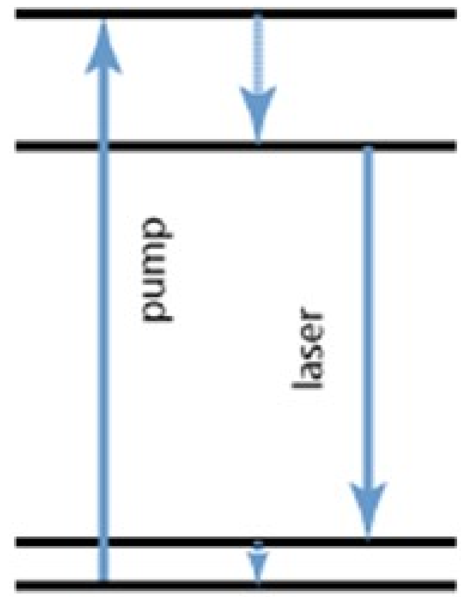


Figure 3

Fig 1: a three-level system, the laser transitions ends on the ground state. Fig.2: a four-level system, the laser transition ends on a level above the ground state, which is quickly depopulated e.g. via phonons. Fig 3: a quasi-three-level system, where the lower laser level has some population in thermal equilibrium.

YouTube Channel Link:
https://www.youtube.com/channel/UC3rdRYA605bdDdsJdEf0oJw/featuredc

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